CBCS SCHEME

USN | | | | | | | | | | | **18MAT31**

Third Semester B.E. Degree Examination, July/August 2021
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find $L[t\,e^{-2t}\sin 4t]$. **(06 Marks)**

b. A periodic function of period $2\pi/\omega$ is defined by $f(t)=\begin{cases} E\sin\omega t, & 0\le t<\pi/\omega \\ 0, & \pi/\omega\le t<2\pi/\omega \end{cases}$. Where E and ω are constants. **(07 Marks)**

c. Solve : $y''(t)+k^2y(t)=0;\ y(0)=0$ and $y'(0)=1$ by Laplace transformation. **(07 Marks)**

2 a. Find : i) $L^{-1}\left\{\dfrac{s^2-3s+4}{s^3}\right\}$ ii) $L^{-1}\left[\mathrm{Cot}^{-1}\left(\dfrac{S}{2}\right)\right]$. **(06 Marks)**

b. Find the inverse Laplace transform of $\dfrac{1}{(s-1)(s^2+1)}$ by using convolution theorem. **(07 Marks)**

c. Express the following function in terms of Heaviside step function and hence find its Laplace transform where $f(t)=\begin{cases} t^2, & 0<t\le 2 \\ 4t, & t>2 \end{cases}$. **(07 Marks)**

3 a. Expand $f(x)=x(2\pi-x)$ as a Fourier series in $[0, 2\pi]$. **(06 Marks)**

b. Obtain Fourier series for the function f(x) given by $f(x)=\begin{cases} 1+\dfrac{2x}{\pi}, & -\pi\le x\le 0 \\ 1-\dfrac{2x}{\pi}, & 0\le x\le\pi \end{cases}$. **(07 Marks)**

c. Find the half range sine series of $f(x)=\dfrac{e^{ax}}{\sinh a\pi}$ in $(0, \pi)$. **(07 Marks)**

4 a. Find the Fourier series expansion of f(x) given by $f(x)=\begin{cases} 1 & 0<x<1 \\ 2 & 1<x<3 \end{cases}$. **(06 Marks)**

b. Find the half range sine series for x^2 in $(0, \pi)$. **(07 Marks)**

c. The values of x and the corresponding values of f(x) over a period T are given below. Show that $f(x)=0.75+0.37\cos\theta+1.004\sin\theta$ where $\theta=\dfrac{2\pi x}{T}$. **(07 Marks)**

x	0	T/6	T/3	T/2	2T/3	5T/6	T
f(x)	1.98	1.30	1.05	1.30	–0.88	–0.25	1.98

5 a. State: i) Initial and final value theorems ii) Find the Z –transform of $\cos\left(\dfrac{n\pi}{2}+\dfrac{\pi}{4}\right)$. **(06 Marks)**

b. Find the complex Fourier transform of the function $f(x)=\begin{cases} 1 & \text{for } |x|\le a \\ 0 & \text{for } |x|>a \end{cases}$. Hence evaluate $\displaystyle\int_0^\infty\left(\frac{\sin x}{x}\right)dx$. **(07 Marks)**

c. Compute the inverse Z-transform of $\dfrac{3z^2+2z}{(5z-1)(5z+2)}$. **(07 Marks)**

6 a. Find the Fourier cosine transform of $f(x) = \begin{cases} x, & 0 < x < 2 \\ 0, & \text{else where} \end{cases}$. (06 Marks)

b. Find the Z-transform of $2n + \sin\frac{n\pi}{4} + 1$. (07 Marks)

c. Solve the difference equation : $u_{n+2} - 3u_{n+1} + 2u_n = 0$, with $u_0 = 0$ and $u_1 = -1$. (07 Marks)

7 a. Find by Taylor's series method the value of y at x = 0.1 to five places of decimals from $\frac{dy}{dx} = x^2y - 1, y(0) = 1$. (06 Marks)

b. Use fourth order Runge-Kutta method to solve $(x + y)\frac{dy}{dx} = 1$, y(0.4) = 1 at x = 0.5 correct to four decimal places. (07 Marks)

c. If $\frac{dy}{dx} = 2e^x - y$, y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.040 and y(0.3) = 2.090, find y(0.4) correct to four decimal places by using Milne's predictor – corrector method and applying corrector formula twice. (07 Marks)

8 a. Using modified Euler's formula compute y(1.1) correct to five decimal places given that $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ and y = 1 at x = 1. [taking h = 0.1]. (06 Marks)

b. Employ Taylor's series method to find y at x = 0.1 and 0.2 correct to four places of decimal. Given $\frac{dy}{dx} - 2y = 3e^x$, y(0) = 0. (07 Marks)

c. Solve the differential equation $y' + y + xy^2 = 0$ with the initial values of y : $y_0 = 1$, $y_1 = 0.9008$, $y_2 = 0.8066$, $y_3 = 0.722$ corresponding to the values of x : $x_0 = 0$, $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$ by computing the value of y corresponding to x = 0.4 applying Adams – Bashforth predictor and corrector formula. (07 Marks)

9 a. Given $y'' - xy' - y = 0$ with the initial conditions y(0) = 1, y′(0) = 0, compute y(0.2) using fourth order Runge-Kutta method. (06 Marks)

b. Derive Euler's equation in the standard form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. (07 Marks)

c. A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is a catenary. (07 Marks)

10 a. Apply Milne's method to compute y(0.8) given that $y'' = 1 - 2yy'$ and the following table of initial values. (07 Marks)

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y′	0	0.1996	0.3937	0.5689

b. Prove that the geodesics on a plane are straight line. (06 Marks)

c. Find the extremal of the functional : $\int_{x_0}^{x_1} (y^2 + y'^2 - 2y\sin x)dx$. (07 Marks)

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